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Directed self-avoiding walks on certain directed random fractals

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Abstract. Using the recently proposed real space renormalisation group method for directed systems, we study the critical behaviour of directed self-avoiding walks (DSAW) on both directed lattice animals (DLA) and directed percolation clusters at threshold p_c (DPC) in two dimensions. The values for the exponent $\nu_\perp$ are found to be $\nu^{\mathrm{DLA}}_{\perp\mathrm{DSAW}} \approx 0.590$ and $\nu^{\mathrm{DPC}}_{\perp\mathrm{DSAW}} \approx 0.528$, both of which are higher than the mean-field value 0.5. It is also shown rigorously that $\nu_\parallel$ remains unchanged; i.e. $\nu^{\mathrm{DLA}}_{\parallel\mathrm{DSAW}} = \nu^{\mathrm{DPC}}_{\parallel\mathrm{DSAW}} = 1$.

1. Introduction

Recently, there has been much interest in studying the statistics of self-avoiding walks (SAW) on fractals. Rammal *et al* (1984) and also Ben-Avraham and Havlin (1984), have studied the critical behaviour of SAW on various finitely ramified non-random fractals. Exact results have been obtained. Comparing the exact results with the simple Flory approximation proposed by Rammal *et al* (1984), they suggested that properties of SAW depend not only on the fractal and fracton dimensions (Mandelbrot 1982, Alexander and Orbach 1982) but also on some other intrinsic aspects of the fractal. For random fractals, Kremer (1981), has studied the SAW properties on diluted diamond lattice at percolation threshold p_c using Monte Carlo methods. He found that the correlation length exponent $\nu^{p_c}_{\mathrm{SAW}}$ has a higher value and can be well approximated by the modified Flory formula $\nu^{p_c}_{\mathrm{SAW}} = 3/(\bar{d}+2)$ where $\bar{d}$ is the fractal dimension of the critical percolation clusters. Kremer's formula has also been found to be a good approximation for SAW in two-dimensional critical percolation clusters by Lam and Zhang (1984). They also used the real space renormalisation group (RSRG) method, to study the SAW properties on two-dimensional lattice animals. In the latter case, Kremers formula is found to be not so good. In spite of these controversies, however, from all the cases studied, it seems to be certain that the correlation length exponent ν_{SAW} always changes to a higher value when the SAW is performed on a fractal instead of its embedding lattice; i.e. pure Euclidean lattice. It also seems to be true that the value of ν^F_{SAW} is larger if the fractal object F has a smaller fractal dimension $\bar{d}_F$. The reasons for the change of ν_{SAW} are given, in the case of critical percolation clusters, by Lyklema and Kremer (1984). However, we believe that it is true for any fractal F with $\bar{d}_F$ smaller than d.

The purpose of this work is to study the properties of directed self-avoiding walks (DSAW) on certain directed random fractals. It is well known that, for a directed system, there are two independent correlation lengths, one parallel and the other perpendicular to the preferred direction ($\xi_\parallel$ and $\xi_\perp$). For the case of DSAW in pure lattices, the corresponding correlation length exponents $\nu_\parallel$ and $\nu_\perp$ are found to be classical when $d \geq 2$; i.e. $\nu_\parallel = 1$ and $\nu_\perp = 0.5$ (Redner and Majid 1983). How the law of statistics will change when DSAW is performed on a fractal with averaged fractal dimension $\bar{d}$ (Kinzel 1983) less than 2 is an interesting question.

Let us consider specifically the statistics of DSAW on directed lattice animals (DLA) and directed percolation clusters at threshold p_c(DPC). For any DLA or DPC configuration α, let $G_N(\boldsymbol{r}; \alpha)$ be the number of DSAW of N steps connecting the origin to site $\boldsymbol{r}$. The quenched mean square end-to-end distances $\langle R_\parallel^2(N)\rangle$ and $\langle R_\perp^2(N)\rangle$ are defined by

$$\langle R_\parallel^2(N)\rangle = \left(\sum_\alpha W(\alpha) \frac{\Sigma_{\boldsymbol{r}} G_N(\boldsymbol{r}; \alpha) r_\parallel^2}{\Sigma_{\boldsymbol{r}} G_N(\boldsymbol{r}; \alpha)}\right) \Big/ \sum_\alpha W(\alpha) \tag{1}$$

and

$$\langle R_\perp^2(N)\rangle = \left(\sum_\alpha W(\alpha) \frac{\Sigma_{\boldsymbol{r}} G_N(\boldsymbol{r}; \alpha) r_\perp^2}{\Sigma_{\boldsymbol{r}} G_N(\boldsymbol{r}; \alpha)}\right) \Big/ \sum_\alpha W(\alpha) \tag{2}$$

where $W(\alpha)$ is the weight for the configuration α and $r_\parallel$ and $r_\perp$ are respectively the parallel and perpendicular distances from site $\boldsymbol{r}$ to the origin projected onto the preferred direction. The large N behaviour of $\langle R_\parallel^2(N)\rangle$ and $\langle R_\perp^2(N)\rangle$ defines the correlation length exponents $\nu_\parallel$ and $\nu_\perp$; i.e. $\langle R_\parallel^2(N)\rangle \sim N^{2\nu_\parallel}$ and $\langle R_\perp^2(N)\rangle \sim N^{2\nu_\perp}$. For DSAW, in any fully directed fractals, either DLA or DPC, it is easy to see that $r_\parallel$ in (1) is always equal to N. Thus we have rigorously $\langle R_\parallel^2(N)\rangle = N^2$ and $\nu_\parallel = 1$. However, to find the value of $\nu_\perp$ is not a trivial task.

In the following, we will use the recently proposed RSRG method for directed systems (Zhang and Yang (1984) hereafter referred to as ZY) to study the value of $\nu_\perp$ for DSAW on DLA and DPC in two dimensions. In this method, for any RG transformation, two effective lengths $S_\parallel(b)$ and $S_\perp(b)$, parallel and perpendicular to the preferred direction, are defined for a given cell of linear size b. The renormalised lattice is constructed from these effective lengths and is deformed from the original lattice. The anisotropic rescaling of these effective lengths give the anisotropic exponents $\nu_\parallel$ and $\nu_\perp$. This method is capable of reproducing the exact results for DSAW and very good results for DLA critical behaviour in two-dimensional Euclidean space (ZY, Yang and Zhang 1984). We believe that this method can also be used to study the critical behaviour of DSAW on DLA and DPC. Before we present the RG calculations, let us make the following remarks. Instead of using the quenched averages defined in (1) and (2), we can also define the annealed averages as

$$\langle R_i^2(N)\rangle = \left(\sum_\alpha W(\alpha) \sum_{\boldsymbol{r}} G_N(\boldsymbol{r}; \alpha) r_i^2\right) \Big/ \left(\sum_\alpha W(\alpha) \sum_{\boldsymbol{r}} G_N(\boldsymbol{r}; \alpha)\right); \qquad i = \parallel \text{ or } \perp. \tag{3}$$

Using the same arguments as given by Harris (1983), it can be shown easily that, at the percolation threshold p_c, the exponent $\nu_\parallel$ and $\nu_\perp$ remain unchanged; i.e. $\nu_\parallel = 1$ and $\nu_\perp = 0.5$. However, in most physical problems, the quenched averages are always more interesting and more difficult to calculate than the annealed averages.

2. DSAW on DLA

Here, we consider specifically DSAW on directed site lattice animals (DSLA) in a square lattice. We construct a two-parameter renormalisation group transformation from an original cell of size $b \times b$ to a renormalised cell of size 1×1. Figure 1 shows the cells for the case of $b=2$ where sites belonging to the original and renormalised cells are marked by full circles. Each site animal in the original and renormalised cells is associated with fugacities K and K' respectively. All the animal configurations which start at the lower-left corner spanning the original cell contribute to K'. To define K', we use both r_0 and r_1 rules as defined by Reynolds *et al* (1980). The r_0 rule requires a connected path which spans the cell either horizontally or vertically. The r_1 rule requires spanning in a particular direction. The recursion relations of K' for $b=2$ and 3 are given in appendix 1.

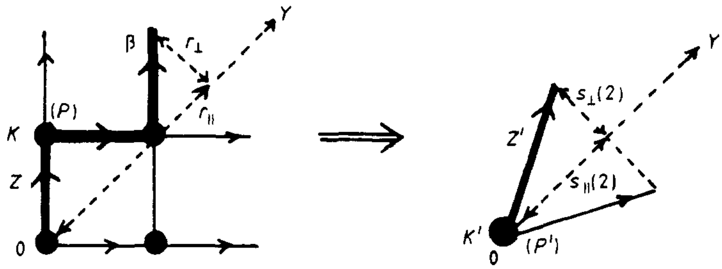

Figure 1. A transformation from an original cell of size 2×2 to a renormalised cell of size 1×1. $r_\parallel$ and $r_\perp$ are shown for a particular DSAW β (bold line). $0Y$ is the preferred direction. The effective lengths $S_\parallel(2)$ and $S_\perp(2)$ form the basic unit of the renormalised lattice.

To study the statistics of DSAW, we associate fugacities Z and Z' to each step of allowed DSAW in the original and renormalised cells respectively. In the original cell, for every spanning site configuration which contributes to K', we count all the allowed DSAW starting from the origin ending on the top edge of the cell. Summing over all possible site configurations which contribute to K' we finally have the corresponding $Z'K'$ of the renormalised cell. It is easy to see that the r_0 and r_1 rules give the same recursion relation of $Z'K'$ although the recursion relations of K' are different. The recursion relations of $Z'K'$ for $b=2$ and 3 are also given in Appendix 1. From both the recursion relations of K' and $Z'K'$, we can obtain the non-trivial fixed point $K^*(b)$ and $Z^*(b)$. The values of $K^*(b)$ together with the cell-to-cell results $K^*(b, b')$ have been given in table 2 of ZY. The values of $Z^*(b)$ and $Z^*(b, b')$ are given in table 1 of this paper.

Since we are studying a directed system, using the RSRG method proposed by ZY, we have to find two effective lengths $S_\parallel(b)$ and $S_\perp(b)$, parallel and perpendicular to the preferred axis, for each original cell of size b. In every configuration α which contributes to K', we project the end-to-end distance $r(\beta;\alpha)$ of a particular spanning DSAW β, which contributes to $Z'K'$, onto two axes, one the preferred direction and the other perpendicular to it. This gives two lengths $r_\parallel(\beta;\alpha)$ and $r_\perp(\beta;\alpha)$ (see figure 1 for $b=2$). We can write the first moment and the second moment definitions of the effective lengths by (ZY)

$$S_i(b) = \left[\left(\sum_\alpha W(\alpha) \frac{\sum_\beta Z^{m(\beta)} r_i(\beta;\alpha)}{\sum_\beta Z^{m(\beta)}}\right) \Big/ \sum_\alpha W(\alpha)\right]_{(k^*, Z^*)}; \qquad i = \parallel \text{ or } \perp \tag{4}$$

Table 1. RG results for DSAW on DSLA in a square lattice, (a) using r_0 rule (b) using r_1 rule.

	b	$b'=1$	2	3	4
(a)					
Z^*	2	1.129			
	3	1.013	0.921		
	4	0.974	0.915	0.908	
	5	0.955	0.911	0.907	0.905
$\nu_\perp$	2	0.637			
	3	0.627	0.604		
	4	0.620	0.597	0.587	
	5	0.616	0.592	0.582	0.576
$\nu_\perp^{(2)}$	2	0.674			
	3	0.662	0.634		
	4	0.654	0.627	0.615	
	5	0.648	0.621	0.610	0.604
$\nu_\parallel$	2	0.959			
	3	0.971	0.986		
	4	0.976	0.988	0.991	
	5	0.980	0.990	0.993	0.995
(b)					
Z^*	2	0.817			
	3	0.842	0.867		
	4	0.856	0.875	0.882	
	5	0.865	0.880	0.885	0.889
$\nu_\perp$	2	0.634			
	3	0.622	0.600		
	4	0.615	0.594	0.584	
	5	0.611	0.589	0.580	0.574
$\nu_\perp^{(2)}$	2	0.675			
	3	0.659	0.631		
	4	0.651	0.624	0.613	
	5	0.645	0.619	0.608	0.602

and

$$S_i^{(2)}(b)=\left[\left(\sum_\alpha W(\alpha)\frac{\sum_\beta Z^{m(\beta)}r_i^2(\beta;\alpha)}{\sum_\beta Z^{m(\beta)}}\right)\Big/\sum_\alpha W(\alpha)\right]^{1/2}_{(k^*,Z^*)};\qquad i=\parallel \text{ or } \perp. \tag{5}$$

In (4) and (5), $W(\alpha)$ is the weight of configuration α which contributes to $K'$and has the expression $K^{n(\alpha)}$ where $n(\alpha)$ is the number of animal sites in the α configuration. $m(\beta)$ in (4) and (5) is the number of steps in the DSAW β. The renormalised lattice is constructed from the effective lengths $S_\parallel(b)$ and $S_\perp(b)$ and is deformed from the original lattice (figure 1). Following the same arguments as given in (6)-(9) of ZY, the DSAW exponents $\nu_\parallel$ and $\nu_\perp$, for the first moment definitions of $S_\parallel(b)$ and $S_\perp(b)$, have the expressions

$$\nu_i(b)=\ln[S_i(b)/S_i(1)]/\ln\lambda(b,1),\qquad i=\parallel \text{ or } \perp \tag{6}$$

where the eigenvalue $\lambda(b, 1)$ is given by $\mathrm{d}Z'/\mathrm{d}Z)_{(K^*, Z^*)}$. Similarly, using the second moment definitions of $S_{\parallel}^{(2)}(b)$ and $S_{\perp}^{(2)}(b)$, we obtain $\nu_{\parallel}^{(2)}(b)$ and $\nu_{\perp}^{(2)}(b)$. We believe that, in the large b limit, both first and second moment definitions of $\nu_{\parallel}$ and $\nu_{\perp}$ will converge to the correct results. In fact, this has been shown to be true when DSAW is performed in a pure square lattice (ZY).

The results of RG calculation for both r_0 and r_1 rules are given in table 1 for $b = 2, 3, 4$ and 5. The values of $\nu_{\parallel}$ are only given for the case of r_0 rule. As expected, the value of $\nu_{\parallel}$ approaches the exact result ($\nu_{\parallel} = 1$) in the large b limit. Similar results are obtained for $\nu_{\parallel}^{(2)}$ in the case of r_1 rule. In order to estimate the limiting value of $\nu_{\perp}(b)$ as b goes to infinite, we use the following extrapolation procedure (Yang and Zhang 1984). We first make the following plausible assumptions. For the cell-to-bond transformation, the effective lengths $S_{\parallel}(b)$, $S_{\perp}(b)$ and the eigenvalue $\lambda(b, 1)$ would behave, in the large b limit, like

$$S_{\parallel}(b) \approx B_1 b(1 + B_2 b^{-y}) \tag{7}$$

$$S_{\perp}(b) \approx C_1 b^{1/\theta}(1 + C_2 b^{-z}) \tag{8}$$

$$\lambda(b, 1) \approx A_1 b^{1/\nu_{\parallel}}(1 + A_2 b^{-x}) \tag{9}$$

where $\theta = \nu_{\parallel}/\nu_{\perp}$ and A_1, A_2, B_1, B_2, C_1 and C_2 are constants. The reasons that we propose (7)-(9) are the following. In the $b \to \infty$ limit, we certainly have $S_{\parallel}(b) \sim b$, this gives (7). The exponents $1/\theta$ and $1/\nu_{\parallel}$ in (8) and (9) are to ensure that correct values of $\nu_{\perp}$ and $\nu_{\parallel}$ are approached as $b \to \infty$. The exponents x, y and z are the corrections to the finite size effect. Equations (7)-(9) are indeed the correct expressions for the case of DSAW on pure square lattice (ZY). Substituting (7)-(9) into (6), we find, to the first three leading terms,

$$\nu_{\perp}^{-1}(b) = \nu_{\perp}^{-1} + e_1(\ln b)^{-1} + e_2(\ln b)^{-2} \tag{10}$$

where e_1 and e_2 are constants independent of b. Fitting the data of table 1 to (10), we find the estimates of $\nu_{\perp}$ and $\nu_{\perp}^{(2)}$ are respectively $\nu_{\perp\mathrm{DSAW}}^{\mathrm{DLM}} \approx 0.578$ and 0.604 for r_0 rule, $\nu_{\perp\mathrm{DSAW}}^{\mathrm{DLA}} \approx 0.576$ and 0.603 for r_1 rule. The best estimate of ν_1 can be determined by the value which gives the best overall fit to the four sets of data simultaneously. From

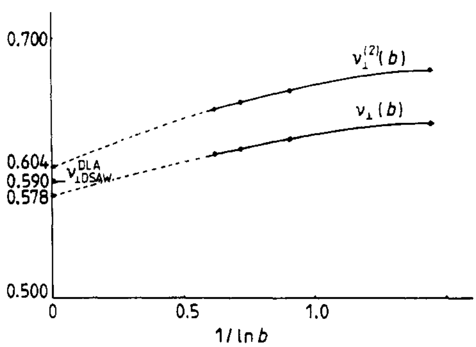

Figure 2. DSAW on DLA. The results of $\nu_{\perp}(b)$ and $\nu_{\perp}^{(2)}(b)$ obtained by using r_0 rule are plotted against $1/\ln b$. The extrapolated values are also shown.

this procedure, we find $\nu_{\perp \mathrm{DSAW}}^{\mathrm{DLA}} \approx 0.590$, $e_1 = -0.213$ and $e_2 = 0.067$. The value 0.590 is indeed much higher than the mean field value 0.5. In order to give an impression of the quality of the results, we plot the values of $\nu_\perp(b)$ and $\nu_\perp^{(2)}(b)$ in figure 2 together with their extrapolated values. Since the results from r_0 and r_1 rules are very close to each other, only the results for the r_0 rule are shown. The best value $\nu_{\perp \mathrm{DSAW}}^{\mathrm{DLA}} \approx 0.590$ obtained from the overall fit is also shown.

3. DSAW on DPC

To study the statistics of DSAW on directed site percolation clusters at threshold p_c(DSPC) in a square lattice, we use a similar method to that described in § 2 with a little modification. Each site belonging to the original cell is occupied with probability p and unoccupied with probability $q = 1 - p$. To define the site occupation probability p' in the renormalised cell, we can use either the r_0 or r_1 rule. As in the case of DSAW on DSLA described in § 2, both r_0 and r_1 rules will lead to very similar results. Here, we will only use the r_1 rule which requires a connected path spanning the cell from one edge to the opposite edge in a particular direction. The recursion relations of p' for $b = 2$ and 3 are given in appendix 2.

In treating the case of DSAW in pure lattice, for each DSAW, ZY have arranged the position of the cells so that the DSAW always pass through the origin of the cells. The effective lengths $S_\parallel(b)$ and $S_\perp(b)$ can be correctly obtained only by doing so. Thus, to study the statistics of DSAW on DPC, among all the possible configurations which contribute to p', we select a set of configurations, A, in which the origin in the original cell is occupied and there exists at least one DSAW connecting the origin to the opposite edge of the cell, because only from those configurations can we obtain the correct statistics of DSAW and the correct effective lengths $S_\parallel(b)$ and $S_\perp(b)$.

Let $f_b(p)$ be the total probability of all the configurations belonging to A. For each configuration α belonging to A, we associate fugacity Z to each step of allowed DSAW β. Two lengths $r_\parallel(\beta;\alpha)$ and $r_\perp(\beta;\alpha)$ are defined in the same way as described in § 2. Thus we have the following expressions for the recursion relation of Z' and the effective lengths $S_\parallel(b)$ and $S_\perp(b)$

$$f_b(p)Z' = \sum_{\alpha\in A} W(\alpha) \sum_{\beta} Z^{m(\beta)} \tag{11}$$

$$S_i(b) = \left[\left(\sum_{\alpha\in A} W(\alpha) \frac{\Sigma_\beta Z^{m(\beta)} r_i(\beta;\alpha)}{\Sigma_\beta Z^{m(\beta)}}\right) \Big/ \sum_{\alpha\in A} W(\alpha)\right]_{(p^*,Z^*)}; \qquad i = \parallel \text{ or } \perp. \tag{12}$$

$W(\alpha)$ is the probability of the configuration α and has the expression $p^{n(\alpha)}q^{t(\alpha)}$ where $n(\alpha)$ and $t(\alpha)$ are respectively the number of occupied and perimeter sites in the α configuration. Clearly, we have $\Sigma_{\alpha\in A} W(\alpha) = f_b(p)$. $m(\beta)$ is the number of steps in the DSAW β. (p^*, Z^*) is the non-trivial fixed point of the recursion relations p' and Z'. The exponents $\nu_\parallel(b)$ and $\nu_\perp(b)$ are still given by (6) with $\lambda(b, 1) = (\mathrm{d}Z'/\mathrm{d}Z)_{(p^*,Z^*)}$. Similarly, the second moment definitions of $S_\parallel^{(2)}(b)$, $S_\perp^{(2)}(b)$, $\nu_\parallel^{(2)}(b)$ and $\nu_\perp^{(2)}(b)$ can also be used. The recursion relations of $f_b(p)Z'$ for $b = 2$ and 3 are given in appendix 2.

The results of RG calculations are given in table 2 for $b = 2, 3, 4$ and 5. In the large b limit, the values of $\nu_\parallel(b)$ also seem to approach the exact result $\nu_\parallel = 1$. Similar results are obtained for $\nu_\parallel^{(2)}$. Fitting the data of table 2 to (10), we find the estimates of $\nu_\perp$

Table 2. RG results for DSAW on DSPC in a square lattice using r_1 rule.

	b	$b'=1$	2	3	4
p^*	2	0.618			
	3	0.648	0.684		
	4	0.664	0.692	0.702	
	5	0.673	0.697	0.705	0.708
Z^*	2	0.674			
	3	0.685	0.686		
	4	0.689	0.688	0.688	
	5	0.692	0.689	0.689	0.689
$\nu_\perp$	2	0.491			
	3	0.491	0.498		
	4	0.492	0.500	0.505	
	5	0.493	0.503	0.508	0.513
$\nu_\perp^{(2)}$	2	0.546			
	3	0.546	0.553		
	4	0.547	0.555	0.559	
	5	0.548	0.557	0.561	0.565
$\nu_\parallel$	2	0.996			
	3	0.993	0.988		
	4	0.992	0.989	0.990	
	5	0.992	0.989	0.990	0.991

and $\nu_\perp^{(2)}$ are respectively $\nu_{\perp \mathrm{DSAW}}^{\mathrm{DPC}} \approx 0.503$ and 0.556. By fitting two sets of data simultaneously, we find $\nu_{\perp \mathrm{DSAW}}^{\mathrm{DPC}} \approx 0.528$ which is again higher than the mean field value 0.5. The values of e_1 and e_2 are respectively $e_1 = 0.074$ and $e_2 = -0.032$.

4. Conclusions and discussion

In this work, the properties of DSAW on DLA and DPC are studied. We have shown rigorously that the DSAW exponent $\nu_\parallel$ always has the value 1 independent of the underlying fractals. To study the possible change of the exponent $\nu_\perp$, we use the RSRG method proposed by ZY extended to the two-parameter case to calculate the values of $\nu_\perp$ for DSAW on DSLA and DSPC in a square lattice. The results are $\nu_{\perp \mathrm{DSAW}}^{\mathrm{DLA}} \approx 0.590$ and $\nu_{\perp \mathrm{DSAW}}^{\mathrm{DPC}} \approx 0.528$. In agreement with our original expectation, the values of $\nu_\perp$ have changed to higher values than the mean field result 0.5. The fact that the value of $\nu_{\perp \mathrm{DSAW}}^{\mathrm{DLA}}$ is larger than that of $\nu_{\perp \mathrm{DSAW}}^{\mathrm{DPC}}$ is also expected. This is because DLA has an averaged fractal dimension $\bar{d}_{\mathrm{DLA}} \approx 2/(0.8+0.5) \approx 1.54$ (Redner and Yang 1982) which is smaller than that of DPC; i.e. $\bar{d}_{\mathrm{DPC}} \approx 2/[0.39(1.734+1.100)] \approx 1.81$ (Kinzel 1983). Finally, we remark that the relative large differences between the values of $\nu_\perp$ and $\nu_\perp^{(2)}$ (tables 1 and 2) indicate that the probability distributions of $r_\perp(\beta;\alpha)$ in (4) and (12) are quite broad. This also explains the slow convergence of the difference between $\nu_\perp$ and $\nu_\perp^{(2)}$ when b becomes large. This is particularly so for the case of DSAW on DPC.

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Appendix 1. Recursion relations for DSAW on DSLA

$b=2$

r_0 rule: $K'=2K^2+3K^3+K^4$

r_1 rule: $K'=K^2+3K^3+K^4$

r_0 and r_1 rules: $K'Z=K^2Z^2+2K^3(Z^2+Z^3)+K^4(Z^2+2Z^3)$

$b=3$

r_0 rule: $K'=2K^3+10K^4+19K^5+20K^6+14K^7+6K^8+K^9$

r_1 rule: $K'=K^3+5K^4+15K^5+19K^6+14K^7+6K^8+K^9$

r_0 and r_1 rules: $K'Z'=K^3Z^3+3K^4(Z^3+Z^4)+K^5(6Z^3+11Z^4+6Z^5)$
$+K^6(9Z^3+19Z^4+16Z^5)+K^7(9Z^3+20Z^4+22Z^5)$
$+K^8(5Z^3+12Z^4+18Z^5)+K^9(Z^3+3Z^4+6Z^5).$

Appendix 2. Recursion relations for DSAW on DSPC using r_1 rule

$b=2$

$$p'=p^2(q+q^2)+3p^3q+p^4$$
$$f_2(p)Z'=p^2q^2Z^2+p^3q(2Z^2+2Z^3)+p^4(Z^2+2Z^3)$$
$$f_2(p)=p^2q^2+3p^3q+p^4$$

$b=3$

$$p'=p^3(q^2+2q^4)+p^4(2q^2+5q^3+4q^4+q^5)+p^5(5q^2+10q^3+9q^4)$$
$$+p^6(q+7q^2+20q^3)+p^7(q+18q^2)+7p^8q+p^9$$
$$f_3(p)Z'=p^3q^4Z^3+p^4q^3Z^3+p^4q^4(3Z^3+2Z^4)+p^4q^5Z^4+p^5q^2Z^5$$
$$+p^5q^3(5Z^3+5Z^4+3Z^5)+p^5q^4(4Z^3+8Z^4+2Z^5)$$
$$+p^6q^2(2Z^3+4Z^4+7Z^5)+p^6q^3(12Z^3+21Z^4+12Z^5)$$
$$+p^7q(Z^4+3Z^5)+p^7q^2(13Z^3+25Z^4+25Z^5)$$
$$+p^8q(6Z^3+14Z^4+21Z^5)+p^9(Z^3+3Z^4+6Z^5)$$
$$f_3(p)=p^3q^4+p^4(q^3+4q^4+q^5)+p^5(q^2+9q^3+9q^4)+p^6(7q^2+20q^3)$$
$$+p^7(q+18q^2)+7p^8q+p^9.$$

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